On a $u_n \in]2,3[\Rightarrow P(u_n) < 0$

$\Rightarrow u_{n+1} - 3 < 0$

$\Rightarrow u_{n+1} < 3$ (1)

- On a $u_{n+1} - 2 = u_n^2 - 5u_n + 7$

Soit $Q(x) = x^2 - 5x + 7$

$D = (-5)^2 - 4 \times 1 \times 7 = -3 < 0$

$\Rightarrow Q(x) > 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow Q(u_n) > 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow u_{n+1} - 2 > 0 \Rightarrow u_{n+1} > 2$ (2)

(1) et (2) $\Rightarrow \quad 2 < u_{n+1} < 3$

Conclusion : $\forall n \in \mathbb{N}, \; 2 < u_n < 3$